

CANDIDATE
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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 **(P3)**

May/June 2018

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.



- 1 Showing all necessary working, solve the equation $\ln(x^4 - 4) = 4 \ln x - \ln 4$, giving your answer correct to 2 decimal places. [4]

[illegible]

2 (i) Given that $\sin(x - 60^\circ) = 3 \cos(x - 45^\circ)$, find the exact value of $\tan x$. [4]

This image shows a blank sheet of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(ii) Hence solve the equation $\sin(x - 60^\circ) = 3 \cos(x - 45^\circ)$, for $0^\circ < x < 360^\circ$. [2]

[illegible]

- 3** A curve has equation $y = \frac{e^{3x}}{\tan \frac{1}{2}x}$. Find the x -coordinates of the stationary points of the curve in the interval $0 < x < \pi$. Give your answers correct to 3 decimal places. [6]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 4** The polynomial $x^4 + 2x^3 + ax + b$, where a and b are constants, is divisible by $x^2 - x + 1$. Find the values of a and b . [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

5 Let $I = \int_{\frac{1}{4}}^{\frac{3}{4}} \sqrt{\left(\frac{x}{1-x}\right)} dx$.

(i) Using the substitution $x = \cos^2 \theta$, show that $I = \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 2 \cos^2 \theta \, d\theta$. [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

[4]

[illegible]

- 6** In a certain chemical reaction the amount, x grams, of a substance is decreasing. The differential equation relating x and t , the time in seconds since the reaction started, is

$$\frac{dx}{dt} = -kx\sqrt{t},$$

where k is a positive constant. It is given that $x = 100$ at the start of the reaction.

- (i) Solve the differential equation, obtaining a relation between x , t and k . [5]

This image shows a full page of a document template designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, and there are no margins, text, or other markings present.

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 7 (i) Showing all working and without using a calculator, solve the equation $z^2 + (2\sqrt{6})z + 8 = 0$, giving your answers in the form $x + iy$, where x and y are real and exact. [3]

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- (ii) Sketch an Argand diagram showing the points representing the roots. [1]

- (iii) The points representing the roots are A and B , and O is the origin. Find angle AOB . [3]

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- (iv) Prove that triangle AOB is equilateral. [1]

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[5]

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(ii) Verify by calculation that a lies between 3 and 3.5. [2]

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(iii) Use an iteration based on the equation in part (i) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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9 Let $f(x) = \frac{12x^2 + 4x - 1}{(x - 1)(3x + 2)}$.

(i) Express $f(x)$ in partial fractions.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 .
[5]

[illegible]

- 10** The point P has position vector $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

- (i) Find the length of the perpendicular from P to l , giving your answer correct to 3 significant figures. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

- (ii) Find the equation of the plane containing l and P , giving your answer in the form $ax + by + cz = d$.
[5]

This image shows a full page of a worksheet designed for handwriting practice. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

[illegible]

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